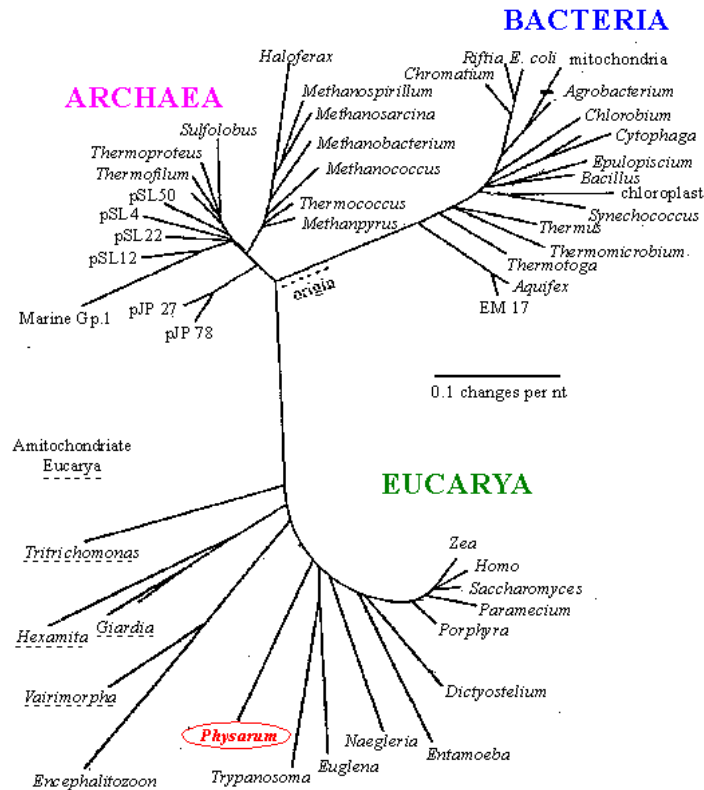

Pairwise Alignment

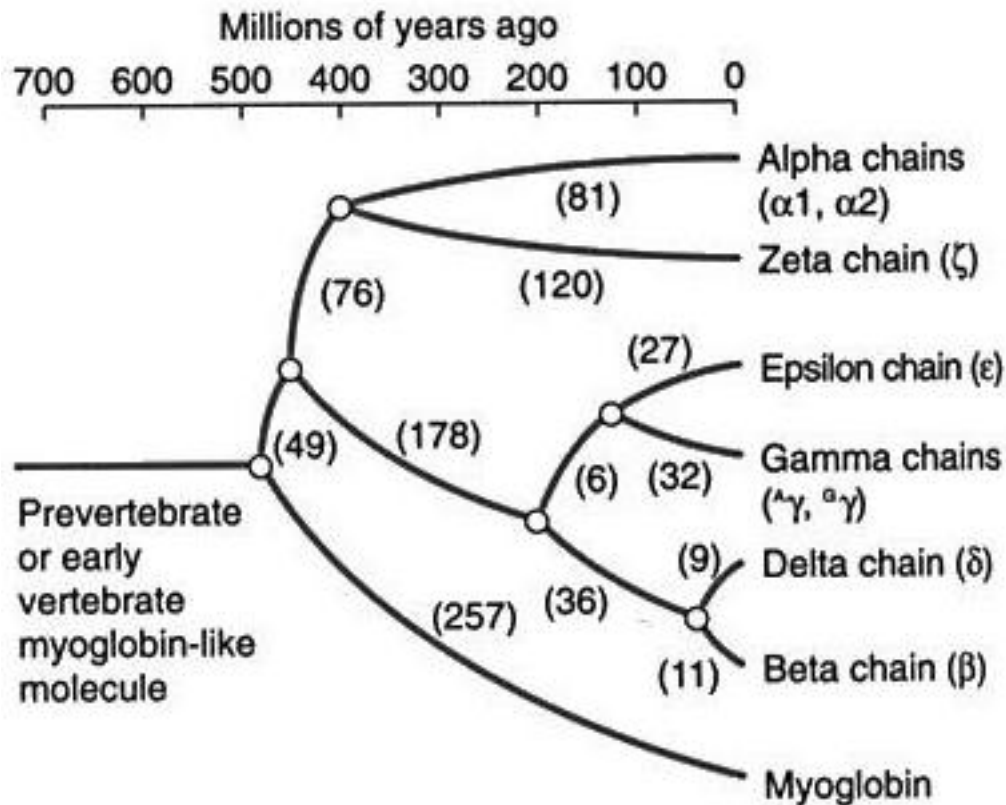
Sequences are related

- Darwin: all organisms are related through descent with modification
- => Sequences are related through descent with modification
- => Similar molecules have similar functions in different organisms



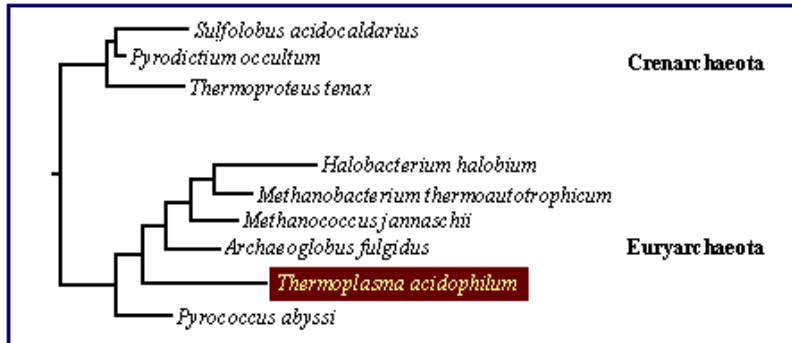
Phylogenetic tree based on
ribosomal RNA:
three domains of life

Sequences are related, II



Phylogenetic tree of globin-type proteins found in humans

Why compare sequences?



- Determination of evolutionary relationships

Protein 1: binds oxygen

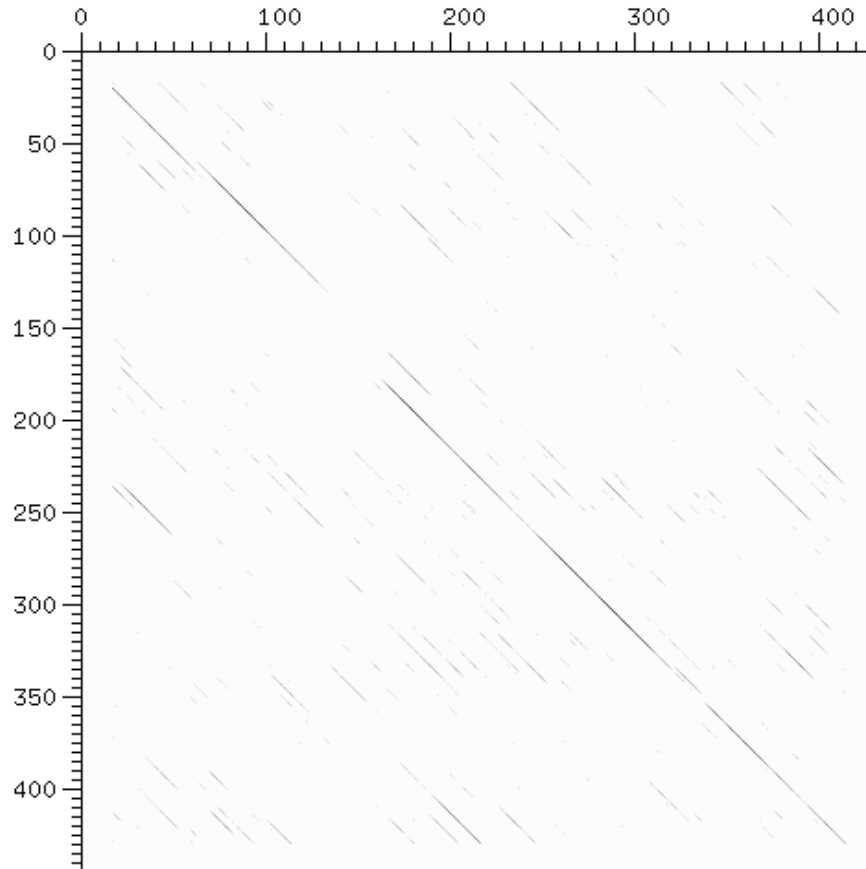


Sequence similarity

Protein 2: binds oxygen ?

- Prediction of protein function and structure (database searches).

Dotplots: visual sequence comparison



1. Place two sequences along axes of plot
2. Place dot at grid points where two sequences have identical residues
2. Diagonals correspond to conserved regions

Pairwise alignments

43.2% identity;

Global alignment score: 374

	10	20	30	40	50	
alpha	V-LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF-DLS-----HGSA					
	: :. : : :. : : :. : : :. : : :. : : :. : : :. : : :.					
beta	VHLTPEEKSAVTALWGKV--NVDEVGGEALGRLLVVYPWTQRFFESFGDLSTPDAMGNP					
	10	20	30	40	50	
	60	70	80	90	100	110
alpha	QVKGHGKKVADALTNVAHVDDMPNALSALSDLHAHKLRVDPVNFKLLSHCLLVTLAAHL					
	.:.:.:. :.:.:. :.:.:. :.:.:. :.:.:. :.:.:. :.:.:. :.					
beta	KVKAHGKKVLGAFSDGLAHLNLTGTFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHF					
	60	70	80	90	100	110
	120	130	140			
alpha	PAEFTPAVHASLDKFLASVSTVLTSKYR					
	: :. :. :. :. :. :. :.					
beta	GKEFTPPVQAAYQKVVAGVANALAHKYH					
	120	130	140			

Pairwise alignment

```
100.000% identity in 3 aa overlap
```

```
SPA
```

```
:::
```

```
SPA
```

Percent identity is not a good measure of alignment quality

Global alignment score: 374

	60	70	80	90	100	110
alpha	QVKGHGKKVADALTNAVAHVDDMPNALSALS DL HAHKLRVDPVNFKLLSHCLLVTLAAHL					
	.:.:.:.:.: :.:.:.:.:.: :.:.:.:.:.: :.:.:.:.:.: :.:.:.:.: :. :.: :.					
beta	KVKAHGKKVLGAFSDGLAHL LD NLKGT FATL SELHCDKLHVDPENFRLLGNVLCVLAH HF					
	60	70	80	90	100	110

	120	130	140
alpha	PAEFTPAVHASLDKFLASVSTVLTSKYR		
	:::: :.		
beta	GKEFTPPVQAAYQKVVAGVANALAHKYH		
	120	130	140

Alignment scores: match vs. mismatch

Simple scoring scheme (too simple in fact...):

Matching amino acids: 5

Mismatch: 0

Scoring example:

K A W S A D V

: : : : :

K D W S A E V

$$\underline{5+0+5+5+5+0+5} = 25$$

Pairwise alignments: conservative substitutions

43.2% identity;

Global alignment score: 374

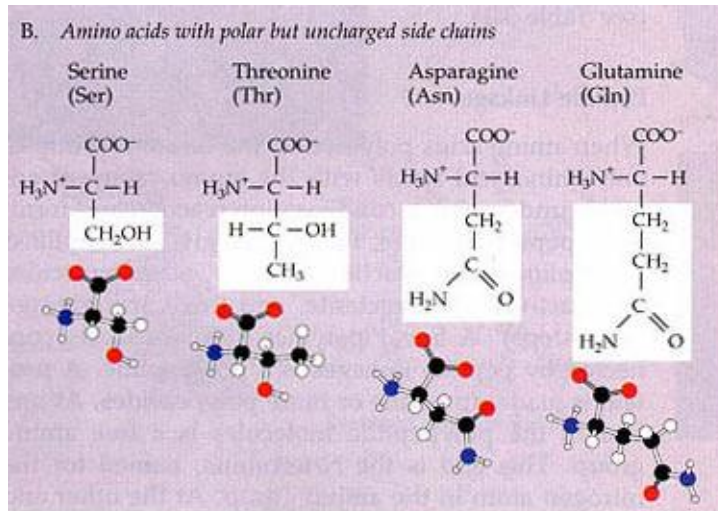
```

              10      20      30      40      50
alpha  V-LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF-DLS-----HGSA
        : : . : : : : : : : : : : : : : : : : : : : : : : : : :
beta   VHLTPEEKSAVTALWGKV--NVDEVGGEALGRLLVVYPWTQRFFESFGDLSTPDVAMGNP
              10      20      30      40      50

              60      70      80      90     100     110
alpha  QVKGHGKKVADALTNAVAHVDDMPNALSALSDDLHAHKLRVDPVNFKLLSHCLLVTLAAHL
        ..... : ..... : ..... : ..... : ..... : .. : :
beta   KVKAHGKKVLGAFSDGLAHLNLDLKGTFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHF
              60      70      80      90     100     110

              120     130     140
alpha  PAEFTPAVHASLDKFLASVSTVLTISKYR
        : : : : : : : : : : : : :
beta   GKEFTPPVQAAYQKVVAGVANALAHKYH
              120     130     140
```

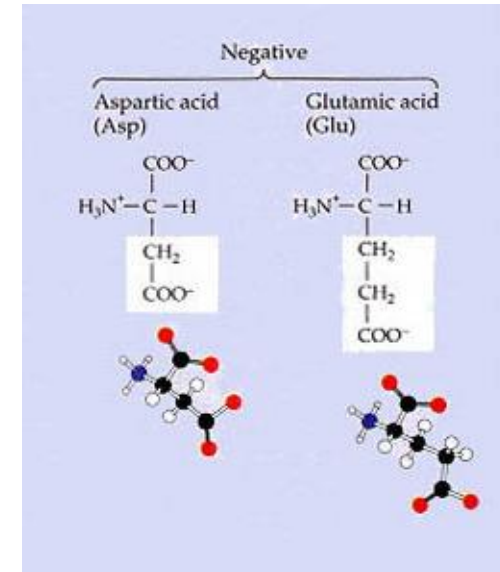
Amino acid properties



Serine (S) and Threonine (T) have similar physicochemical properties

=> Substitution of S/T or E/D occurs relatively often during evolution

=> Substitution of S/T or E/D should result in scores that are only moderately lower than identities



Aspartic acid (D) and Glutamic acid (E) have similar properties

Protein substitution matrices

BLOSUM50 matrix:

A	5																			
R	-2	7																		
N	-1	-1	7																	
D	-2	-2	2	8																
C	-1	-4	-2	-4	13															
Q	-1	1	0	0	-3	7														
E	-1	0	0	2	-3	2	6													
G	0	-3	0	-1	-3	-2	-3	8												
H	-2	0	1	-1	-3	1	0	-2	10											
I	-1	-4	-3	-4	-2	-3	-4	-4	-4	5										
L	-2	-3	-4	-4	-2	-2	-3	-4	-3	2	5									
K	-1	3	0	-1	-3	2	1	-2	0	-3	-3	6								
M	-1	-2	-2	-4	-2	0	-2	-3	-1	2	3	-2	7							
F	-3	-3	-4	-5	-2	-4	-3	-4	-1	0	1	-4	0	8						
P	-1	-3	-2	-1	-4	-1	-1	-2	-2	-3	-4	-1	-3	-4	10					
S	1	-1	1	0	-1	0	-1	0	-1	-3	-3	0	-2	-3	-1	5				
T	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	2	5			
W	-3	-3	-4	-5	-5	-1	-3	-3	-3	-3	-2	-3	-1	1	-4	-4	-3	15		
Y	-2	-1	-2	-3	-3	-1	-2	-3	2	-1	-1	-2	0	4	-3	-2	-2	2	8	
V	0	3	3	4	1	3	3	4	4	4	1	3	1	1	3	2	0	3	1	5
	A	R	N	D	C	Q	E	G	H	I	L	K	M	F	P	S	T	W	Y	V

- Positive scores on diagonal (identities)

- Similar residues get higher (positive) scores

- Dissimilar residues get smaller (negative) scores

Pairwise alignments: insertions/deletions

43.2% identity;

Global alignment score: 374

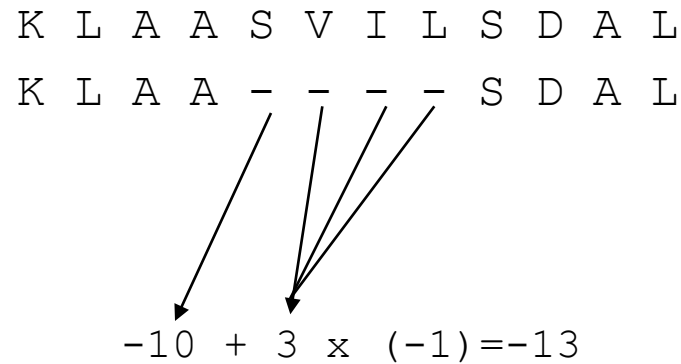
```

      10      20      30      40      50
alpha  V-LSPADKTNVKAAWGKVGAHAGEYGAEALERMFLSFPTTKTYFPHF-DLS-----HGSA
      :  ::  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :
beta   VHLTPEEKSAVTALWGKV--NVDEVGGEALGRLLVVYPWTQRFFESFGDLSTPDAVMGNP
      10      20      30      40      50

      60      70      80      90     100     110
alpha  QVKGHGKKVADALTNAVAHVDDMPNALSALSDDLHAHKLRVDPVNFKLLSHCLLVTLAAHL
      :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :
beta   KVKAHGKKVLGAFSDGLAHLNLKGTFFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHF
      60      70      80      90     100     110

      120     130     140
alpha  PAEFTPAVHASLDKFLASVSTVLTISKYR
      :  :  :  :  :  :  :  :  :  :  :  :  :  :  :  :
beta   GKEFTPPVQAAYQKVVAGVANALAHKYH
      120     130     140
```

Alignment scores: insertions/deletions



Affine gap penalties:

Multiple insertions/deletions may be one evolutionary event =>

Separate penalties for **gap opening** and **gap elongation**

Handout

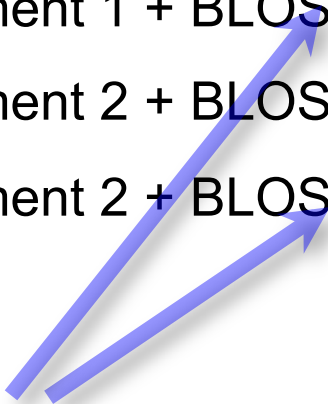
Compute 4 alignment scores: two different alignments using two different alignment matrices (and the same gap penalty system)

Score 1: Alignment 1 + BLOSUM-50 matrix + gaps

Score 2: Alignment 1 + BLOSUM-Trp matrix + gaps

Score 3: Alignment 2 + BLOSUM-50 matrix + gaps

Score 4: Alignment 2 + BLOSUM-Trp matrix + gaps



Note: fake matrix constructed for pedagogic purposes.

Handout: summary of results

	Alignment 1	Alignment 2
BLOSUM-50	38	51
BLOSUM-Trp	118	91

Protein substitution matrices

BLOSUM50 matrix:

- Positive scores on diagonal (identities)
- Similar residues get higher (positive) scores
- Dissimilar residues get smaller (negative) scores

A	5																			
R	-2	7																		
N	-1	-1	7																	
D	-2	-2	2	8																
C	-1	-4	-2	-4	13															
Q	-1	1	0	0	-3	7														
E	-1	0	0	2	-3	2	6													
G	0	-3	0	-1	-3	-2	-3	8												
H	-2	0	1	-1	-3	1	0	-2	10											
I	-1	-4	-3	-4	-2	-3	-4	-4	-4	5										
L	-2	-3	-4	-4	-2	-2	-3	-4	-3	2	5									
K	-1	3	0	-1	-3	2	1	-2	0	-3	-3	6								
M	-1	-2	-2	-4	-2	0	-2	-3	-1	2	3	-2	7							
F	-3	-3	-4	-5	-2	-4	-3	-4	-1	0	1	-4	0	8						
P	-1	-3	-2	-1	-4	-1	-1	-2	-2	-3	-4	-1	-3	-4	10					
S	1	-1	1	0	-1	0	-1	0	-1	-3	-3	0	-2	-3	-1	5				
T	0	-1	0	-1	-1	-1	-1	-2	-2	-1	-1	-1	-1	-2	-1	2	5			
W	-3	-3	-4	-5	-5	-1	-3	-3	-3	-3	-2	-3	-1	1	-4	-4	-3	15		
Y	-2	-1	-2	-3	-3	-1	-2	-3	2	-1	-1	-2	0	4	-3	-2	-2	2	8	
V	0	-3	-3	-4	-1	-3	-3	-4	-4	4	1	-3	1	-1	-3	-2	0	-3	-1	5
	A	R	N	D	C	Q	E	G	H	I	L	K	M	F	P	S	T	W	Y	V

Protein substitution matrices: different types

- Identity matrix
(match vs. mismatch)
- Chemical properties matrix
(use knowledge of physicochemical properties to design matrix)
- ➔ • Empirical matrices
(based on observed pair-frequencies in hand-made alignments)
 - PAM series
 - BLOSUM series
 - Gonnet

Estimation of the BLOSUM 50 matrix

- BLOSUM matrices are computed based on gap-free alignments in the so-called BLOCKS database. BLOSUM 50 is computed by comparing sequences that are less than 50% identical. BLOSUM 80 is computed from sequences less than 80% identical, etc.
- All pairs of sequences in a block are compared, and the observed pair frequencies are noted (e.g., A aligned with A makes up 1.5% of all pairs. A aligned with C makes up 0.01% of all pairs, etc.)
- Expected pair frequencies are computed from single amino acid frequencies. (e.g, $f_{A,C} = f_A \times f_C = 7\% \times 3\% = 0.21\%$).
- For each amino acid pair the substitution scores are essentially computed as:

ID	FIBRONECTIN_2; BLOCK
COG9_CANFA	GNSAGEPCVFPFIFLGKQYSTCTREGRGDGHLWCATT
COG9_RABIT	GNADGAPCHFPFTFEGRSYACTTDGRSDGMAWCSTT
FA12_HUMAN	LTVTGEPCHFPPQYHRQLYHKCTHKGRPGPQPWCATT
HGFA_HUMAN	LTEDGRPCRFPFRYGGRLHACTSEGSABRKWCATTH
MANR_HUMAN	GNANGATCAFPFKFENKWDYADCTSAGRSDGWLWCSTT
MPRI_MOUSE	ETDDGEPVFPFPIYKGSYDECVLGRAKLWCSKTAN
PB1_PIG	AITSDDKCVFPFIYKGNLYFDCTLHDSTYYWCSVTY
SFP1_BOVIN	ELPEDEECVFPFVYRNRKHFDCVHGSLSFPWCSLDAD
SFP3_BOVIN	AETKDNKCVFPFIYGNKKYFDCTLHGSLSFLWCSLDAD
SFP4_BOVIN	AVFEGPACAFPFYKGGKYYMCTRKNSVLLWCSLDTE
SP1_HORSE	AATDYAKCAFPFVYRGQTYDRCTTDGSLFRISWCSVT
COG2_CHICK	GNSEGAPCVFPFIFLGKNDYDCTSAGRNDGKLWCAST
COG2_HUMAN	GNSEGAPCVFPFTFLGNKYESCTSAGRSDGKMWCAAT
COG2_MOUSE	GNSEGAPCVFPFTFLGNKYESCTSAGRNDGKVMCAAT
COG2_RABIT	GNSEGAPCVFPFTFLGNKYESCTSAGRSDGKMWCAAT
COG2_RAT	GNSEGAPCVFPFTFLGNKYESCTSAGRNDGKVMCAAT
COG9_BOVIN	GNADGKPCVFPFTFQGRYSACTSDGRSDGYRWCATT
COG9_HUMAN	GNADGKPCQFPFIFQGSYSACTTDGRSDGYRWCATT
COG9_MOUSE	GNGEGKPCVFPFIFEGRSYSACTTKGRSDGYRWCATT
COG9_RAT	GNGDGKPCVFPFIFEGHSYSACTTKGRSDGYRWCATT
FINC_BOVIN	GNSNGALCHFPFLYNNHNYTDCTSEGRRDNMKWCGTT
FINC_HUMAN	GNSNGALCHFPFLYNNHNYTDCTSEGRRDNMKWCGTT
FINC_RAT	GNSNGALCHFPFLYSNRNYSYDCTSEGRRDNMKWCGTT
MPRI_BOVIN	ETEDGEPVFPFVFNGKSYEECVVESRRLWCATTAN
MPRI_HUMAN	ETDDGVPCVFPFIENGKSYEECIIESRAKLWCSTTAD
PA2R_BOVIN	GNAHGTPCMFPFQYNQWWHHECTREGREDNLLWCATT
PA2R_RABIT	GNAHGTPCMFPFQYNHQQWWHHECTREGRQDDSLWCATT

$$\log \frac{\text{Pair-freq(obs)}}{\text{Pair-freq(expected)}} \quad S_{A,C} = \log \frac{0.01\%}{0.21\%} = -1.3$$

Pairwise alignment

Optimal alignment:

alignment having the highest possible score given a substitution matrix and a set of gap penalties

So:

best alignment can be found by exhaustively searching all possible alignments, scoring each of them and choosing the one with the highest score?

The problem:

How many possible alignments are there?

ACG	AC-G	--ACG	-A-CG
ACG	ACG-	AC-G-	A-CG-
-ACG	AC-G	--ACG	...
ACG-	A-CG	A-CG-	
-ACG	AC-G	--ACG	
AC-G	-ACG	AC--G	
-ACG	ACG-	--ACG	
A-CG	AC-G	A-C-G	
A-CG	ACG-	--ACG	
ACG-	A-CG	A--CG	
A-CG	ACG-	-A-CG	
AC-G	-ACG	ACG--	
A-CG	--ACG	-A-CG	
-ACG	ACG--	AC-G-	

Pairwise alignment: the problem

The number of possible pairwise alignments increases explosively with the length of the sequences:

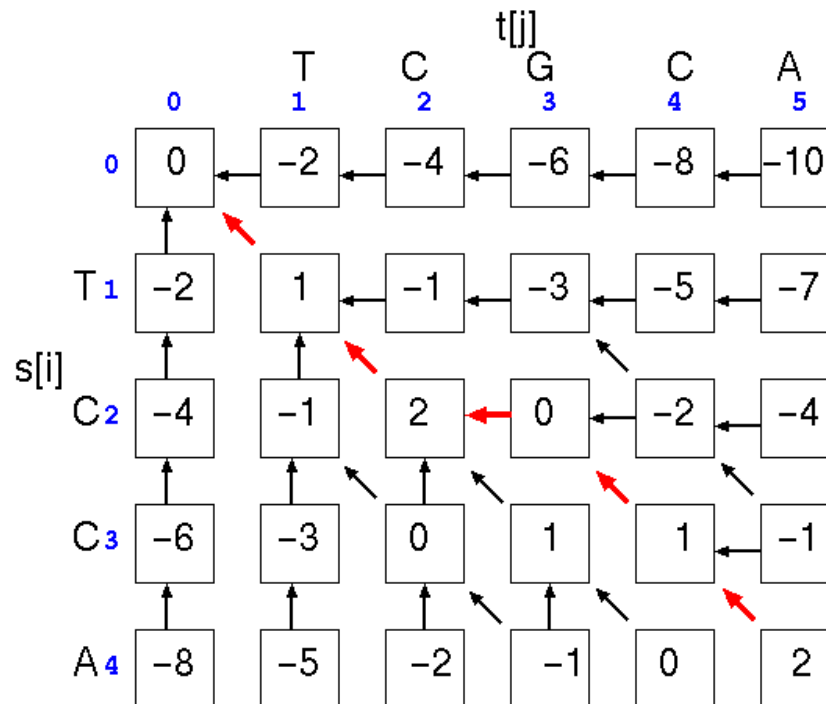


Two protein sequences of length 100 amino acids can be aligned in approximately 10^{60} different ways

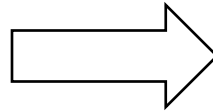
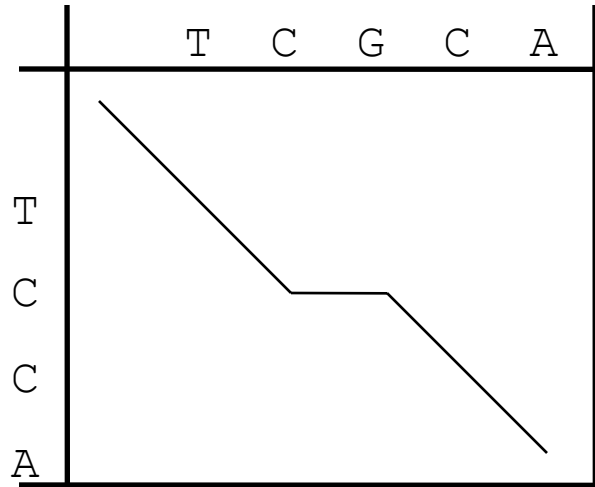
Time needed to test all possibilities is same order of magnitude as the entire lifetime of the universe.

Pairwise alignment: the solution

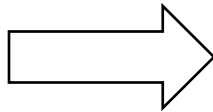
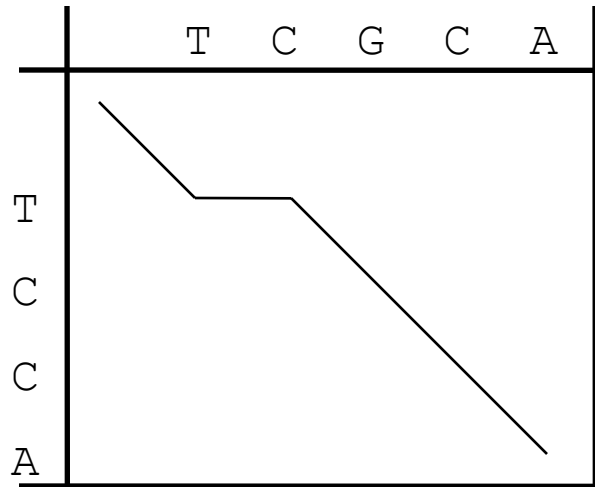
”Dynamic programming”
(the Needleman-Wunsch algorithm)



Alignment depicted as path in matrix

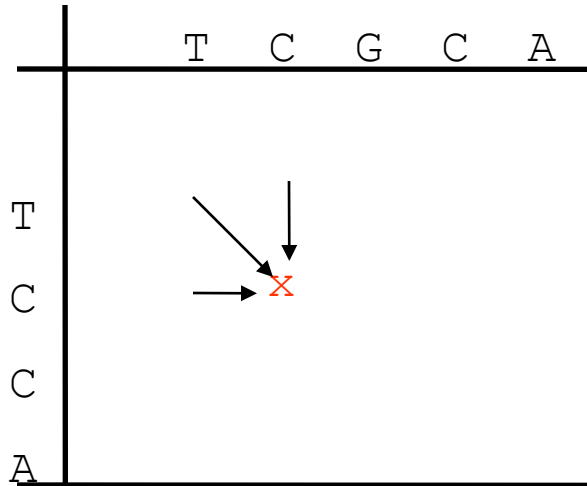


TCGCA
TC-CA



TCGCA
T-CCA

Dynamic programming: computation of scores



Any given point in matrix can only be reached from three possible previous positions (you cannot “align backwards”).

=> Best scoring alignment ending in any given point in the matrix can be found by choosing the highest scoring of the three possibilities.

Dynamic programming: computation of scores

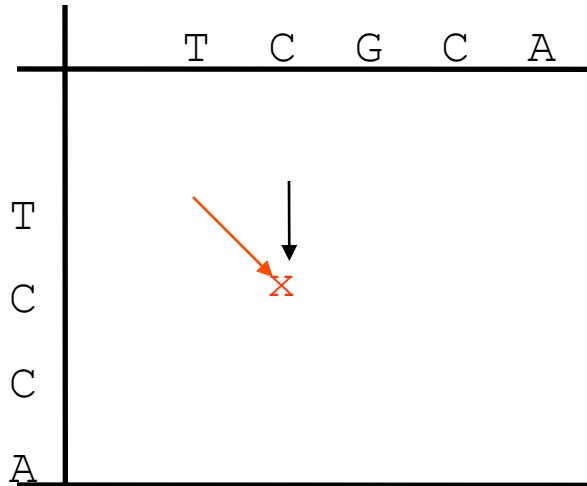
	T	C	G	C	A
T					
C		x			
C					
A					

Any given point in matrix can only be reached from three possible positions (you cannot “align backwards”).

=> Best scoring alignment ending in any given point in the matrix can be found by choosing the highest scoring of the three possibilities.

$$\text{score}(x,y) = \max \left\{ \begin{array}{l} \text{score}(x,y-1) - \text{gap-penalty} \\ \text{score}(x-1,y) - \text{gap-penalty} \\ \text{score}(x-1,y-1) + \text{match/mismatch} \end{array} \right.$$

Dynamic programming: computation of scores

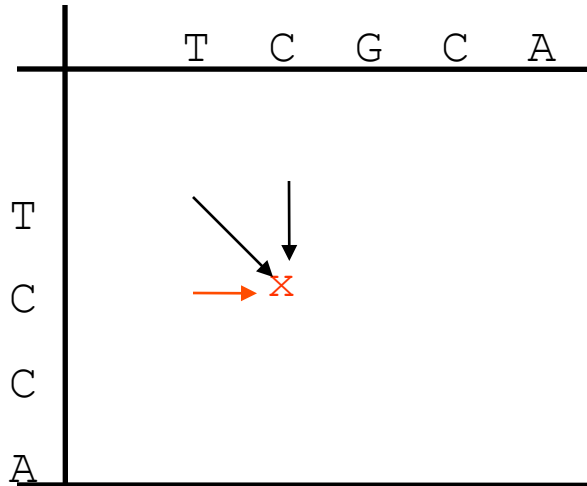


Any given point in matrix can only be reached from three possible positions (you cannot “align backwards”).

=> Best scoring alignment ending in any given point in the matrix can be found by choosing the highest scoring of the three possibilities.

$$\text{score}(x,y) = \max \left\{ \begin{array}{l} \text{score}(x,y-1) - \text{gap-penalty} \\ \text{score}(x-1,y-1) + \text{substitution-score}(x,y) \end{array} \right.$$

Dynamic programming: computation of scores

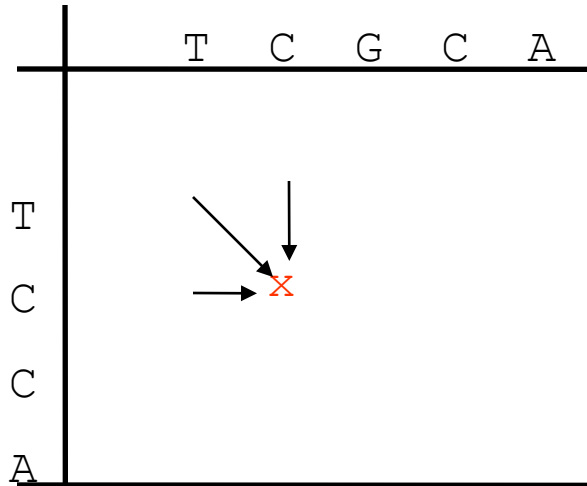


Any given point in matrix can only be reached from three possible positions (you cannot “align backwards”).

=> Best scoring alignment ending in any given point in the matrix can be found by choosing the highest scoring of the three possibilities.

$$\text{score}(x,y) = \max \begin{cases} \text{score}(x,y-1) - \text{gap-penalty} \\ \text{score}(x-1,y-1) + \text{substitution-score}(x,y) \\ \text{score}(x-1,y) - \text{gap-penalty} \end{cases}$$

Dynamic programming: computation of scores



Any given point in matrix can only be reached from three possible positions (you cannot “align backwards”).

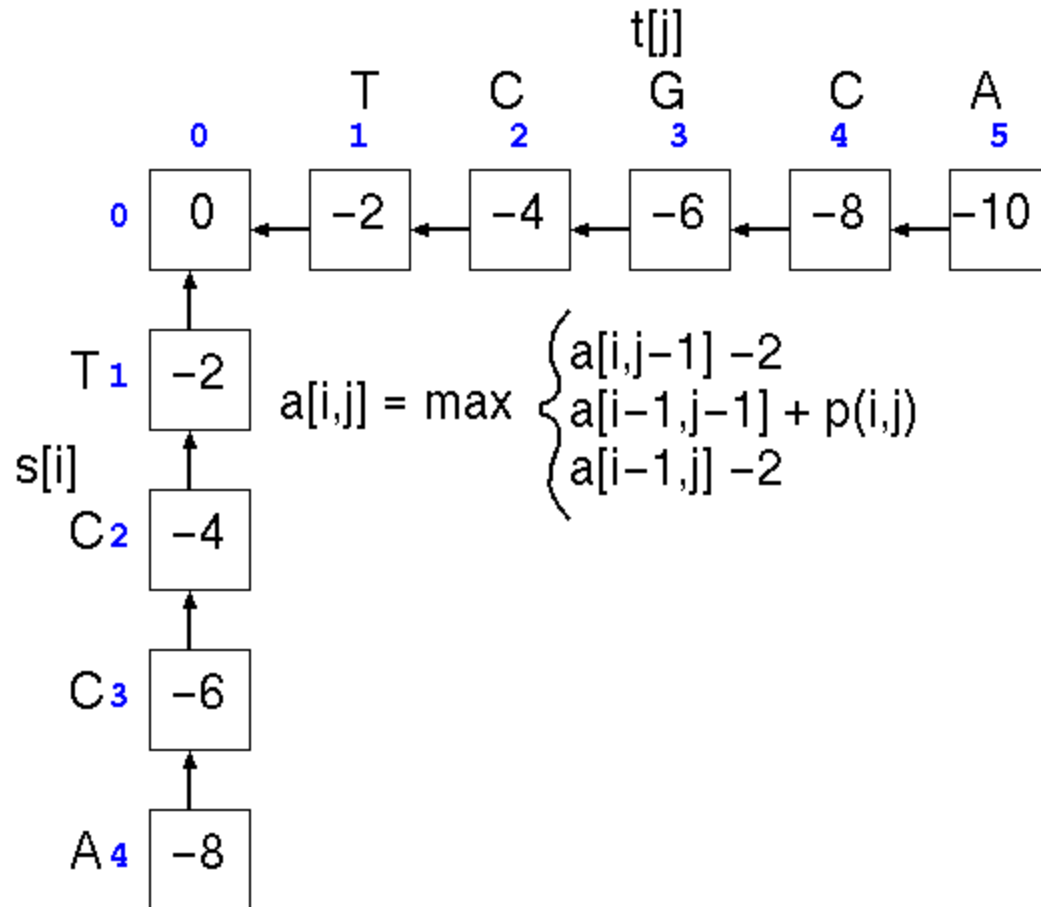
=> Best scoring alignment ending in any given point in the matrix can be found by choosing the highest scoring of the three possibilities.

Each new score is found by choosing the maximum of three possibilities.
For each square in matrix: keep track of where best score came from.

Fill in scores one row at a time, starting in upper left corner of matrix, ending in lower right corner.

$$\text{score}(x,y) = \max \begin{cases} \text{score}(x,y-1) - \text{gap-penalty} \\ \text{score}(x-1,y-1) + \text{substitution-score}(x,y) \\ \text{score}(x-1,y) - \text{gap-penalty} \end{cases}$$

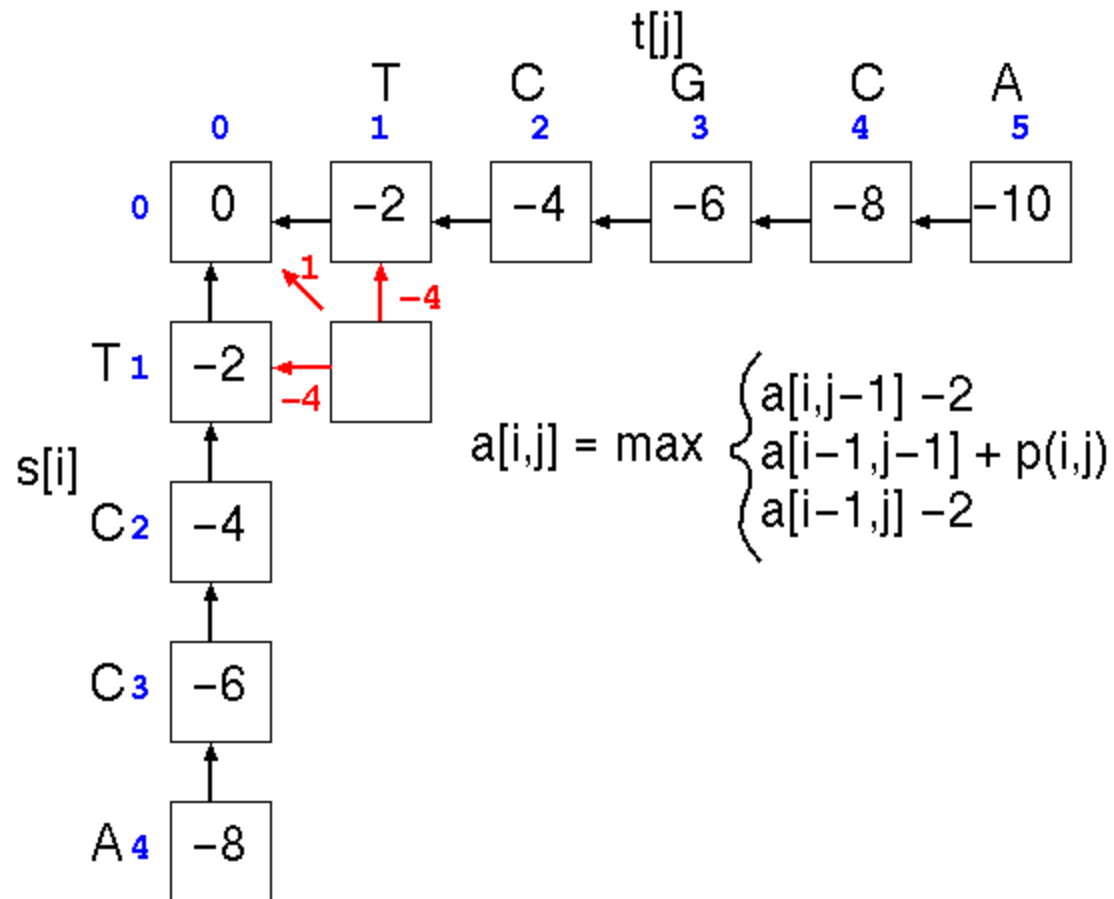
Dynamic programming: example



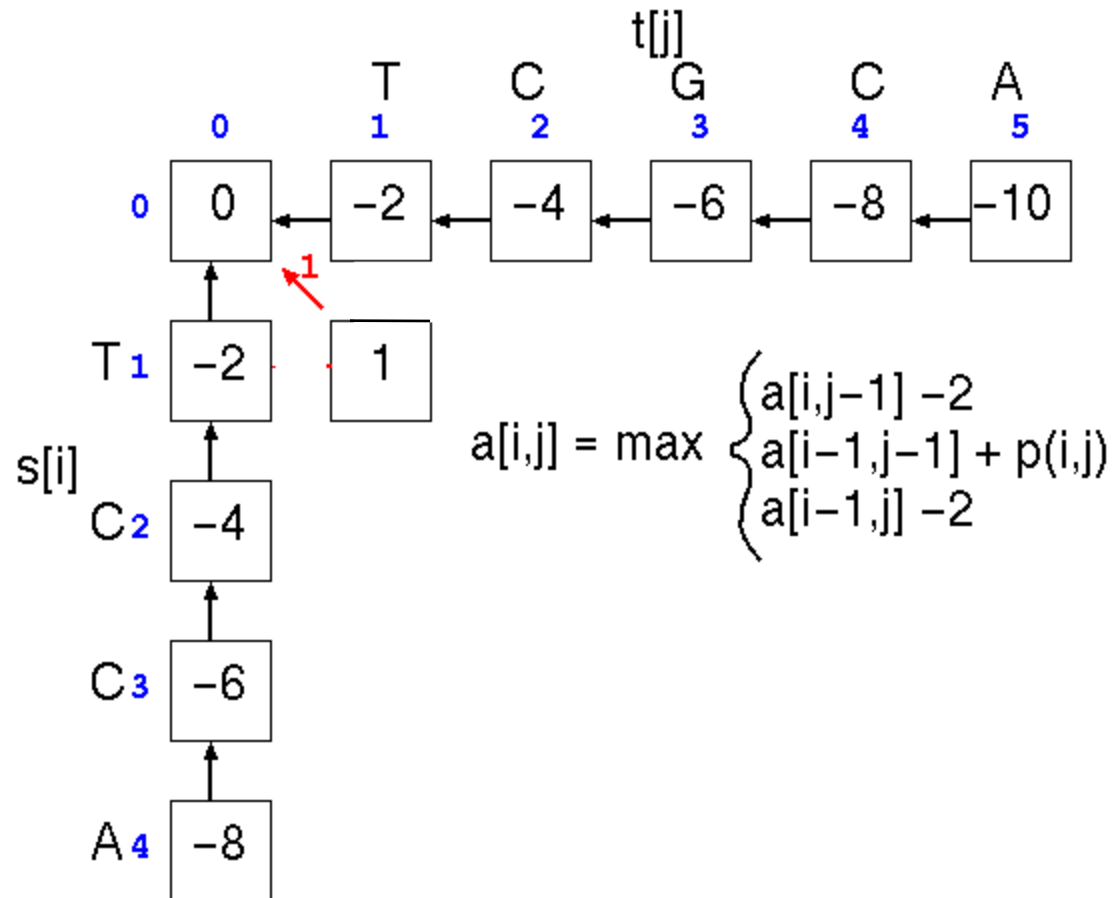
	A	C	G	T
A	1	-1	-1	-1
C	-1	1	-1	-1
G	-1	-1	1	-1
T	-1	-1	-1	1

Gaps: -2

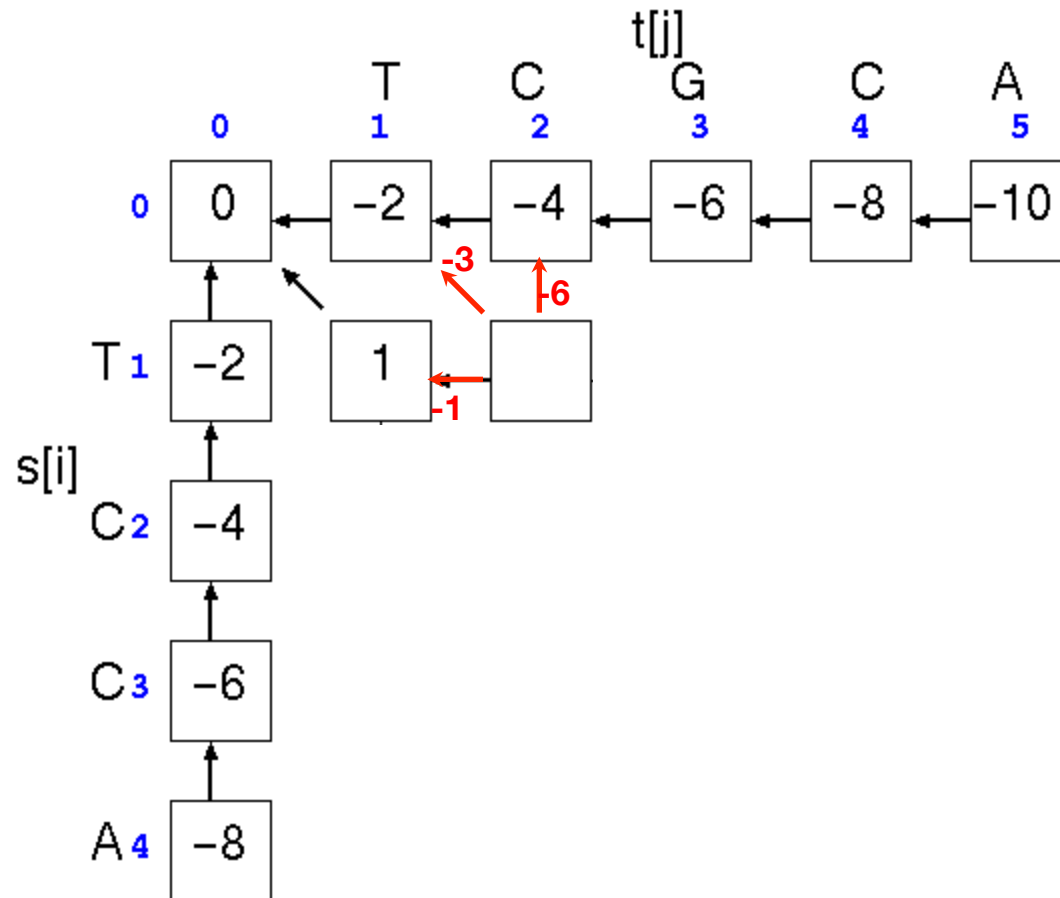
Dynamic programming: example



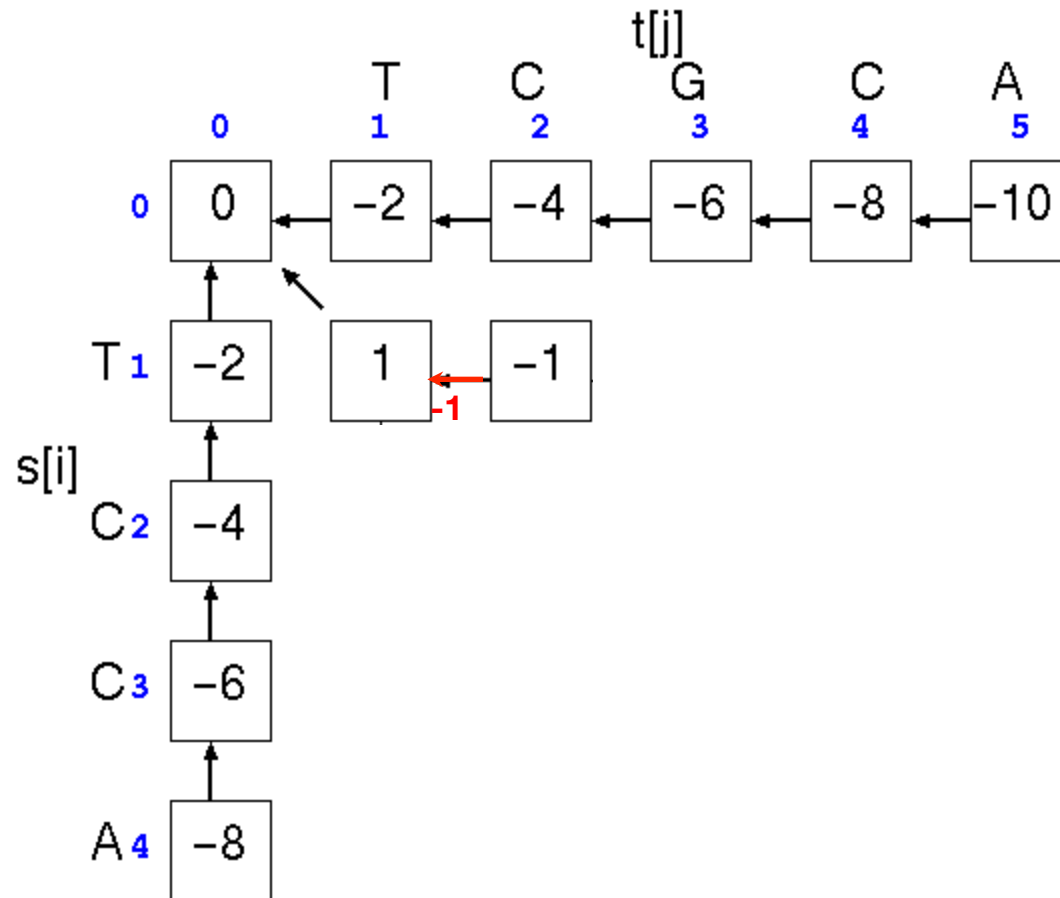
Dynamic programming: example



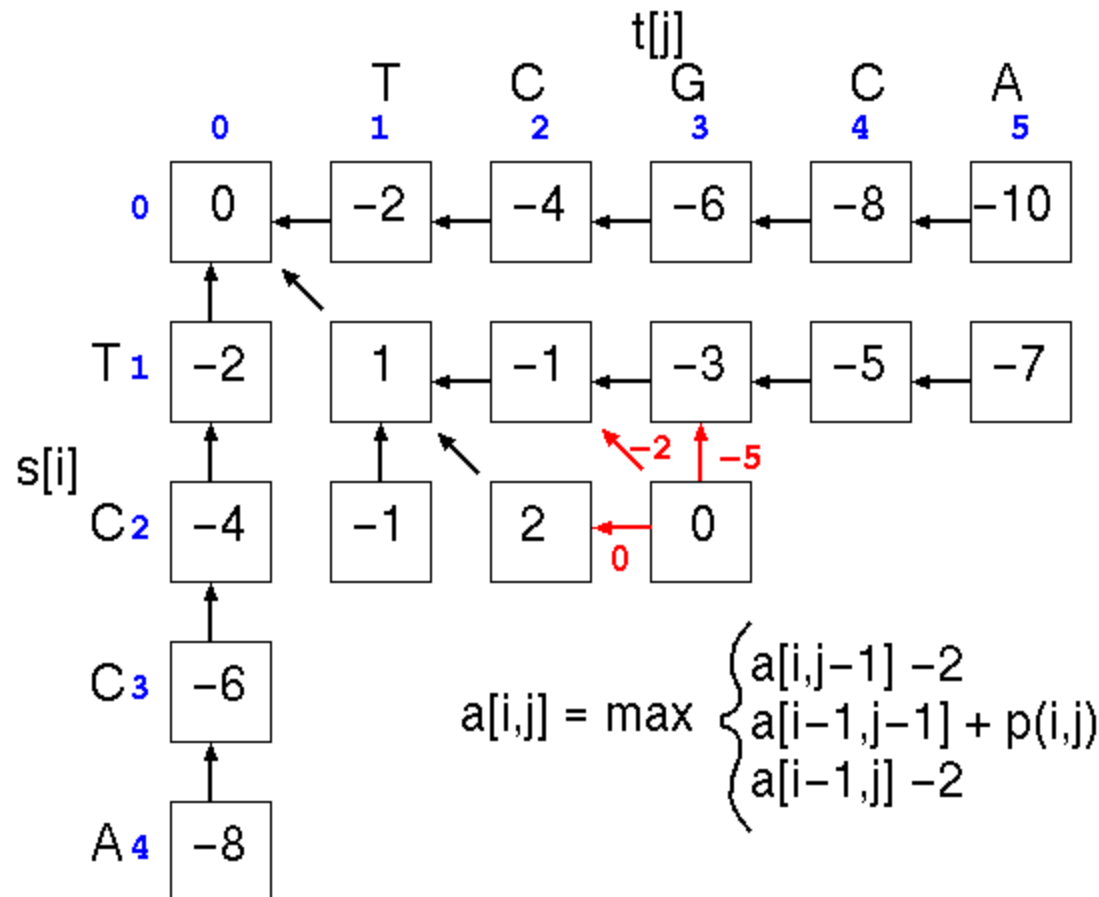
Dynamic programming: example



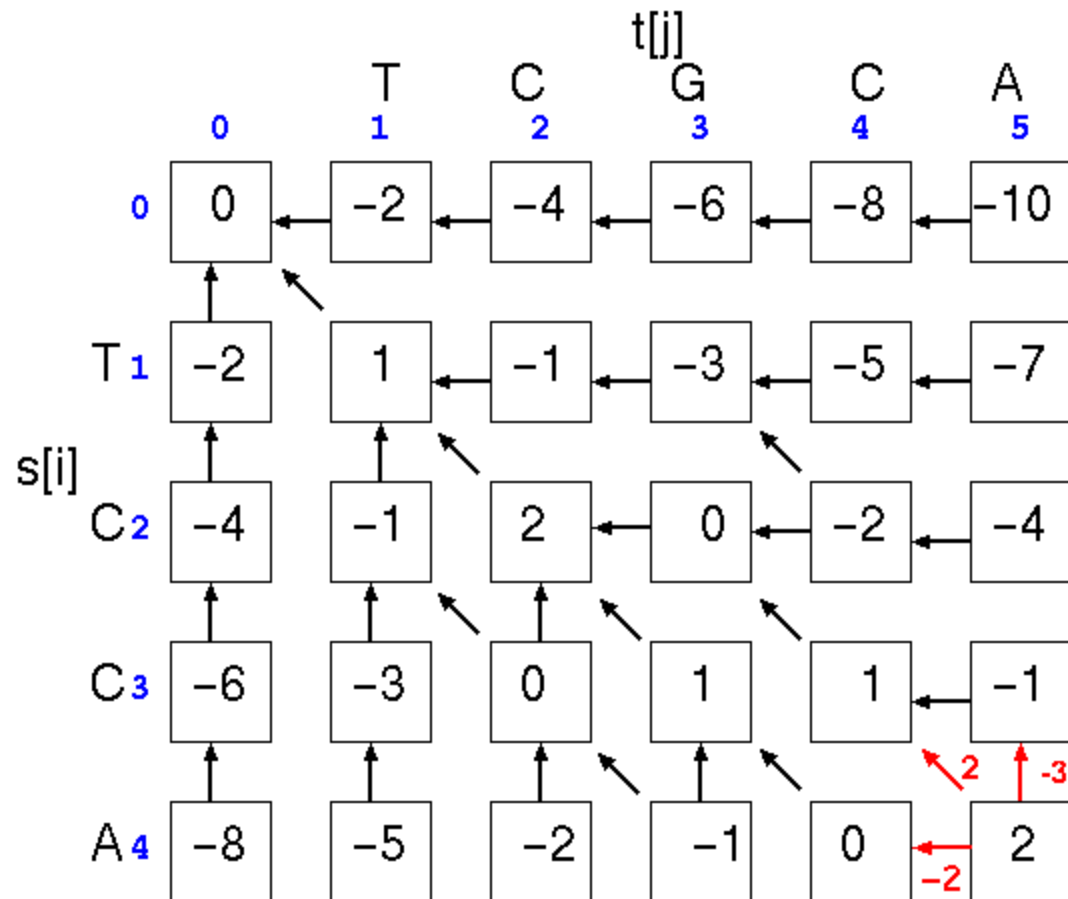
Dynamic programming: example



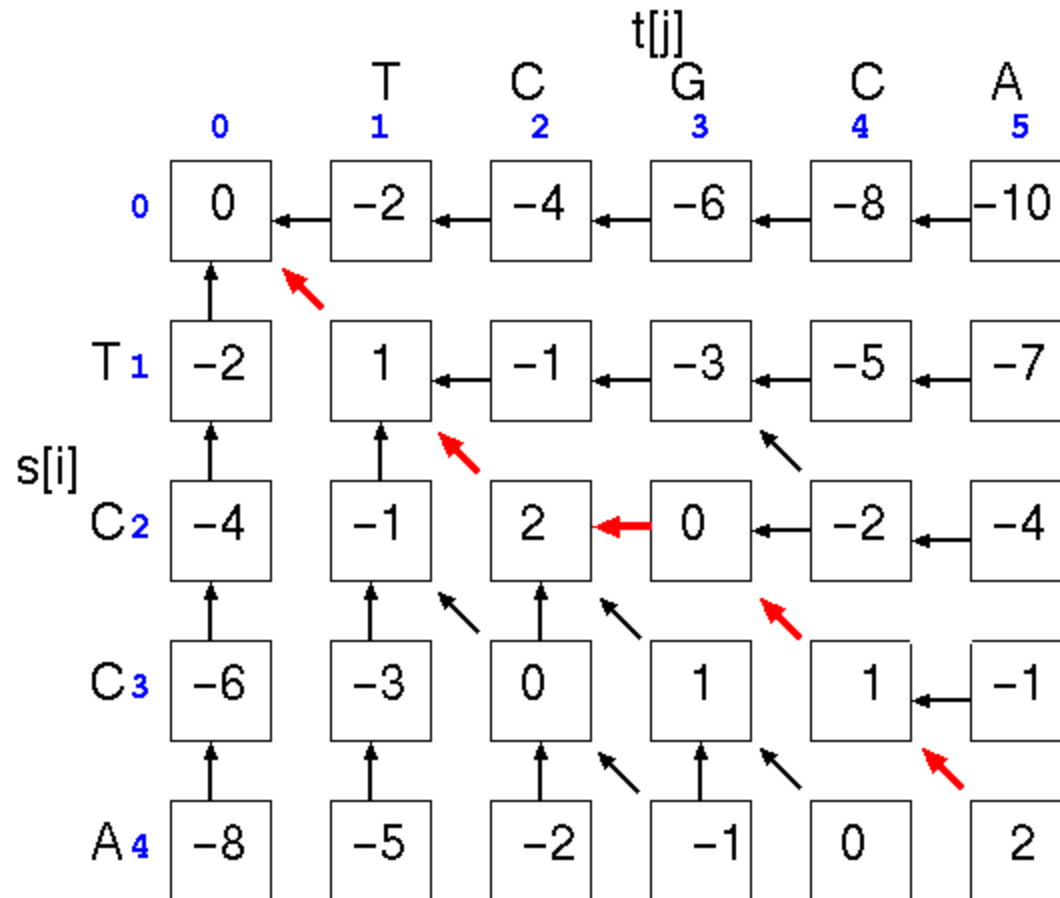
Dynamic programming: example



Dynamic programming: example



Dynamic programming: example



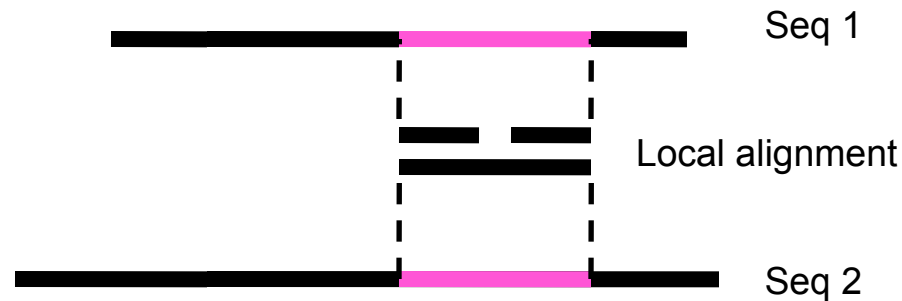
$$\begin{array}{cccccc}
 & T & C & G & C & A \\
 & : & : & & : & : \\
 & T & C & - & C & A \\
 \hline
 & 1 & +1 & -2 & +1 & +1 & = & \underline{2}
 \end{array}$$

Global versus local alignments

Global alignment: align full length of both sequences. (The “Needleman-Wunsch” algorithm).



Local alignment: find best partial alignment of two sequences (the “Smith-Waterman” algorithm).



Local alignment overview

- The recursive formula is changed by adding a fourth possibility: zero. This means local alignment scores are never negative.

$$\text{score}(x,y) = \max \begin{cases} \text{score}(x,y-1) - \text{gap-penalty} \\ \text{score}(x-1,y-1) + \text{substitution-score}(x,y) \\ \text{score}(x-1,y) - \text{gap-penalty} \\ 0 \end{cases}$$

- Trace-back is started at the highest value rather than in lower right corner
- Trace-back is stopped as soon as a zero is encountered

Local alignment: example

		H	E	A	G	A	W	G	H	E	E
	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	5	0	5	0	0	0	0	0
W	0	0	0	0	2	0	20	12	4	0	0
H	0	10	2	0	0	0	12	18	22	14	6
E	0	2	16	8	0	0	4	10	18	28	20
A	0	0	8	21	13	5	0	4	10	20	27
E	0	0	6	13	18	12	4	0	4	16	26

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Substitution matrices and sequence similarity

- Substitution matrices come as series of matrices calculated for different degrees of sequence similarity (different evolutionary distances).
- "Hard" matrices are designed for similar sequences
 - Hard matrices are designated by high numbers in the BLOSUM series (e.g., BLOSUM80)
 - Hard matrices yield short, highly conserved alignments
- "Soft" matrices are designed for less similar sequences
 - Soft matrices have low BLOSUM values (45)
 - Soft matrices yield longer, less well conserved alignments

Alignments: things to keep in mind

“Optimal alignment” means “having the highest possible score, given substitution matrix and set of gap penalties”.

This is NOT necessarily the biologically most meaningful alignment.

Specifically, the underlying assumptions are often wrong: substitutions are not equally frequent at all positions, affine gap penalties do not model insertion/deletion well, etc.

Pairwise alignment programs always produce an alignment - even when it does not make sense to align sequences.